

# Extended Abstract

## Are centenarians getting older? Cohort comparison of lifespan after age 100 in Denmark since 1970

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### Abstract

Previous work on trends in the age at death of cohorts of Scandinavian, namely Swedish centenarians suggested that there is no upward trend in the mean ages at death although the maximum ages at death were observed to be increasing. Using Danish data the mean age at death for centenarians has also been stable at approximately 102 years with the maximum ages at death for successive cohorts trending upwards. We unify the analysis of these two seemingly disparate phenomena by using quantile regression. We show that while there is no trend over most centenarian ages at death, the top 4% oldest cohorts have been experiencing longer lifespans.

## 1 Introduction

Globally, life expectancy at birth has been steadily increasing and more than doubled in the last two centuries (Riley, 2001). In some industrialised countries this has occurred at a pace of about 3 months per year for females for 160 years (Oeppen and Vaupel, 2002) and currently indicates no signs of slowdown (White, 2002). Whereas this increase in life expectancy was driven by reductions in infant mortality in earlier times, from around the 1950s the main driver has been reductions in mortality at older ages (Vaupel et al., 1998; Jeune and Vaupel, 1995; Vaupel, 1997).

One direct consequence of this is the rapid increase in - and accumulation of - the number of nonagenarians, centenarians and supercentenarians (Vaupel and Jeune, 1995; Maier et al., 2010). This phenomenon is particularly evident in the low mortality countries of Europe (Robine and Paccaud, 2005; Thatcher, 2001), Japan (Robine and Saito, 2003; Robine et al., 2003) and North America (Krach and Velkoff, 1999) where the number of centenarians has been roughly doubling every decade since the 1960's (Vaupel and Jeune, 1995).

The maximum observed individual lifespan has been increasing steadily for over a century-and-a-half in the case of Sweden where high quality data permits such observation (Wilmoth and Robine, 2003; Wilmoth et al., 2000). This pattern is not limited to Sweden. In France, Japan, England & Wales and the United States where data and record keeping is of a high quality, national demographic statistics also suggest a secular rise in the maximum age at death (Wilmoth and Lundström, 1996).

Drefahl et al. (2012) showed that for Sweden, the average age at death has been nearly constant over time. They noted that, “Despite the stable mortality pattern amongst Swedish centenarians, we saw a continued increase in the maximum age at death... Our observations demonstrate that the survival pattern after age 100 has not contributed to the rise of the maximum age at death.”

What does the Danish data suggest? Is there also stationary in the average age at death of centenarians over time along with a clear secular increase in maximum ages at death? How do we account for the increasing number of centenarians?

We demonstrate that the survival pattern after age 100 has contributed to the rise in the maximum age at death, in the particular case of Denmark over the time period of our data. However, attempting to study trends in centenarian survival by examining only averages can be problematic. Looking at only the centre of the conditional age at death distribution may cause important features at other points in the distribution to be missed. We show that by adopting a quantile regression approach we can broaden our scope by examining the full conditional distribution and so gain greater insights into Danish centenarian survival.

## 2 Data and Methods

This study is based on a Danish data set maintained by the Department of Epidemiology, Biostatistics and Biodemography of University of Southern Denmark.

The data comes from the Danish *Central Person Register*, (Civil Registration System) and consists of all individuals who have attained age 100 in Denmark, having been born between 1870 and 1904 inclusive.

The standard ordinary least squares (OLS) linear regression model for one predictor variable is given by,

$$Y = \mathbf{X}^T \boldsymbol{\beta} + e, \quad E(e) = 0$$

Thus  $E(Y|\mathbf{X} = x) = \mathbf{x}^T \boldsymbol{\beta}$ , where  $\boldsymbol{\beta}$  measures the marginal change in the mean of  $Y$  due to a marginal change in  $x$ . Typically Gaussian errors with constant variance and zero mean is assumed. The regression coefficients are solved by ordinary least squares and the model specification is primarily designed to predict the mean of the conditional distribution.

[Koenker and Bassett \(1978\)](#) developed the general theory of quantile regression which extends the general regression framework to conditional quantiles of the response variable, such as the 99th percentile. Quantile regression is particularly useful when the rate of change in the conditional quantile, expressed by the regression coefficients, depends on the quantile. For a random variable  $Y$  with probability distribution function

$$F(y) = P(Y \leq y)$$

the  $\tau$ th quantile of  $Y$  is defined as the inverse function

$$Q_\tau(Y) = \inf\{y : F(y) \geq \tau\}$$

where  $0 < \tau < 1$  is the quantile level. E.g.  $Q_{0.5}$  is the median,  $Q_{0.75}$  is the third quartile or 75th percentile

Suppose  $Y$  is the response variable, and  $\mathbf{X}$  is the  $p$ -dimensional predictor. Let  $F_Y(y|\mathbf{X} = x) = P(Y \leq y|\mathbf{X} = x)$  be the conditional cumulative distribution function (CDF) of  $Y$  given  $\mathbf{X} = x$ . Then the  $\tau$ th conditional quantile of  $Y$  is defined as

$$Q_\tau(Y|\mathbf{X} = x) = \inf\{y : F(y) \geq \tau\}$$

This can be extended to the General Linear quantile regression model:

$$Q_\tau(Y|\mathbf{X} = x) = \mathbf{X}^T \boldsymbol{\beta}(\tau), \quad 0 < \tau < 1,$$

where  $\boldsymbol{\beta}(\tau) = (\beta_1(\tau), \dots, \beta_p(\tau))^T$  is the quantile coefficient that may depend on  $\tau$  and represents the marginal change in the  $\tau$ th quantile due to the marginal change in  $x$ .

Whereas the linear regression coefficients are solved by minimising least squares,

$$\hat{\boldsymbol{\beta}} = \arg \min_{\boldsymbol{\beta} \in \mathbf{R}^p} \sum_{i=1}^n (y_i - \mathbf{x}_i^T \boldsymbol{\beta})$$

the  $\tau$ th regression quantile of the linear conditional quantile function  $Q_\tau(Y|\mathbf{X} = x)$  is estimated by minimizing a weighted sum of the absolute deviations,

$$\hat{\beta}(\tau) = \arg \min_{\beta \in \mathbf{R}^p} \sum_{i=1}^n \rho_\tau(y_i - \mathbf{x}_i^T \beta)^2$$

for any quantile  $\tau \in (0, 1)$ . For example the case  $\tau = 0.5$  corresponds to median regression.

In our particular case we fit, for any choice of quantile  $\tau$ , the following model:

$$Q_\tau(\text{Age at death}|\text{Birth Year}) = \beta_0(\tau) + \beta_1(\tau) * \text{Birth Year}$$

The model coefficients are estimated using the `quantreg` package of the R statistical software program (R Core Team, 2016).

### 3 Preliminary Results

Linear (OLS) regression predicts only the conditional mean of the age at death distribution and assumes that the variability of the distribution over the predicted conditional means is the same over all ages. On the other hand, quantile regression explores the entire range of the conditional age at death distribution – not just the mean age at death but any arbitrary point in the age at death distribution. By considering the conditional age at death distribution at different points, we therefore gain a more complete understanding of how age at death changes with cohort. Figure 1 presents the data fitted with several quantile regressions: 5th to 95th percentiles in increments of 5 plus the 96th, 97th and 99th percentiles. It is immediately obvious that the effect of birth year on age at death is much greater at the highest ages at death judging by the increasing slope of the fitted lines.

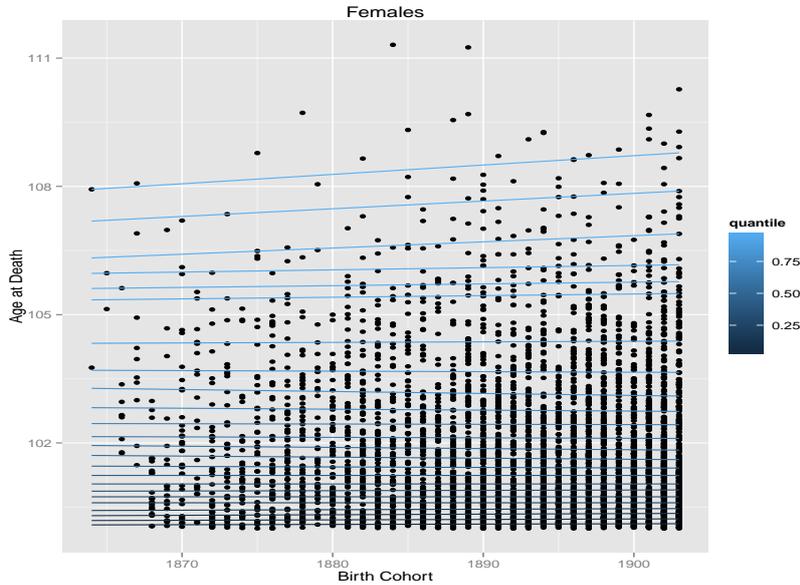


Figure 1: Fitted regression quantiles:0.05 to 0.95 in steps of 0.05 plus 0.96, 0.97 and 0.99

For example the age at death increases by about 0.041 years per one unit increase in birth year at the 99th percentile but only by 0.025 years per year at the 98th percentile. At the 85th percentile and below there is no statistically significant change.

Of course the increase in maximal ages could simply be a statistical artifact of the increased numbers of persons reaching the higher ages. Ongoing work involves distinguishing whether the increased ages are truly driven by increased longevity or is driven by higher numbers.

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